A LaTeX example of equation alignment

(1) Prove that for all $n \in \mathbb{Z}^+$,

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}.$$

Solution:

We prove this by induction on n. Let $P(n)$ be the statement that

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}.$$

We check the base case $n = 1$ is true. $P(1)$ is the statement that $1 = \frac{1 \cdot (1 \cdot 1)}{2}$. But this is true as the right-hand side is equal to 1.

For the induction step, assume that $P(k)$ is true for some $k \in \mathbb{Z}^+$. We will show that this implies $P(k+1)$ is true. $P(k)$ is the statement

$$\sum_{i=1}^{k} i = \frac{k(k+1)}{2}.$$

Adding $k+1$ to both sides and manipulating, we obtain:

$$
\begin{eqnarray*}
1+2+3+\ldots+k+(k+1) &=& \frac{k(k+1)}{2}+k+1\\
&=& (k+1)\left(\frac{k}{2}+1\right)\\
&=& (k+1)\left(\frac{k+2}{2}\right)\\
&=& \frac{(k+1)(k+2)}{2}.
\end{eqnarray*}
$$

Comparing the left-hand side to the final right-hand-side expression, we see the desired statement $P(k+1)$. Thus the induction step is proved and by the principle of induction, $P(n)$ is true for all n as required.

LaTeX Remark. I have aligned the equations above using the environment

```
\begin{eqnarray*}
1+2+3+\ldots+k+(k+1) &=& \frac{k(k+1)}{2}+k+1\\
&=& (k+1)\left(\frac{k}{2}+1\right)\\
&=& (k+1)\left(\frac{k+2}{2}\right)\\
&=& \frac{(k+1)(k+2)}{2}.
\end{eqnarray*}
```

For this environment, I can align equations around an "=" sign by using two ampersands. The double backslash tells the compiler to start a new line. The asterisks suppress equation numbering. I could get numbered, aligned equations by removing the asterisks.

Another way to align equations is:

```
\begin{align*}
1+2+3+\ldots+k+(k+1) &= \frac{k(k+1)}{2}+k+1\\
&= (k+1)\left(\frac{k}{2}+1\right)\\
&= (k+1)\left(\frac{k+2}{2}\right)\\
&= \frac{(k+1)(k+2)}{2}
\end{align*}
```

The two versions look a little different but each gets the job done (although "align" can sometimes be buggy).

A very versatile way to align a lot of stuff and get more than three columns is to use an array:

```
\[
\begin{array}{rrcl}
& 1+2+3+\ldots+k+(k+1) &=& \frac{k(k+1)}{2}+k+1\\
\implies & 1+2+3+\ldots+k+(k+1) &=& (k+1)\left(\frac{k}{2}+1\right)\\
\implies & 1+2+3+\ldots+k+(k+1) &=& (k+1)\left(\frac{k+2}{2}\right)\\
\implies & 1+2+3+\ldots+k+(k+1) &=& \frac{(k+1)(k+2)}{2}
\end{array}
\]
```

Notice I needed to already declare math mode around the array. I also have to think about how many columns in the array and declare whether they are aligned left, right or center. There are four columns, indicated to the compiler by the three ampersands, and the alignment within columns is the "rrcl", you can see. This code spits out:

$$
\begin{array}{rrcl}
& 1+2+3+\ldots+k+(k+1) & = & \frac{k(k+1)}{2}+k+1\\
\implies & 1+2+3+\ldots+k+(k+1) & = & (k+1)\left(\frac{k}{2}+1\right)\\
\implies & 1+2+3+\ldots+k+(k+1) & = & (k+1)\left(\frac{k+2}{2}\right)\\
\implies & 1+2+3+\ldots+k+(k+1) & = & \frac{(k+1)(k+2)}{2}.
\end{array}
$$

Notice this has changed the fractions in a way I probably didn't want. Array has made everything "inline" maths fonts. To fix this, I can go back and force the fractions to be "displaystyle" (check the code):

$$
\begin{array}{rrcl}
& 1+2+3+\ldots+k+(k+1) & = & \dfrac{k(k+1)}{2}+k+1\\
\implies & 1+2+3+\ldots+k+(k+1) & = & (k+1)\left(\dfrac{k}{2}+1\right)\\
\implies & 1+2+3+\ldots+k+(k+1) & = & (k+1)\left(\dfrac{k+2}{2}\right)\\
\implies & 1+2+3+\ldots+k+(k+1) & = & \dfrac{(k+1)(k+2)}{2}.
\end{array}
$$

Now I have a new problem that the lines aren't far enough apart! I can fix this by adding a stretching command (check the code – you can mess around with the stretching variables to get it how you like):

$$
\begin{array}{rrcl}
& 1+2+3+\ldots+k+(k+1) & = & \dfrac{k(k+1)}{2}+k+1\\
\implies & 1+2+3+\ldots+k+(k+1) & = & (k+1)\left(\dfrac{k}{2}+1\right)\\
\implies & 1+2+3+\ldots+k+(k+1) & = & (k+1)\left(\dfrac{k+2}{2}\right)\\
\implies & 1+2+3+\ldots+k+(k+1) & = & \dfrac{(k+1)(k+2)}{2}.
\end{array}
$$